2010 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE

GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2010

学科試験　　問題

EXAMINATION QUESTIONS

（高等専門学校留学生）

COLLEGE OF TECHNOLOGY STUDENTS

数　　学

MATHEMATICS

注意　☆試験時間は 60 分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

Q 1 The quadratic function which takes the value 41 at $x = -2$ and the value 20 at $x = 5$ and is minimized at $x = 2$ is

$$y = \boxed{A}\,x^2 - \boxed{B}\,x + \boxed{C}.$$

The minimum value of this function is $\boxed{D}$.

Q 2   Consider the integral expression in x

$$P = x^3 + x^2 + ax + 1,$$

where a is a rational number.

At $a = \boxed{\textbf{A}}$ the value of P is a rational number for any x which satisfies the equation $x^2 + 2x - 2 = 0$, and in this case the value of P is $\boxed{\textbf{B}}$.

Q 3 For each of $\boxed{\text{A}} \sim \boxed{\text{D}}$ in the following statements, choose the most appropriate expression from among $\textcircled{0} \sim \textcircled{9}$ below.

Consider two conditions $x^2 - 3x - 10 < 0$ and $|x - 2| < a$ on a real number x, where a is a positive real number.

(i) A necessary and sufficient condition for $x^2 - 3x - 10 < 0$ is that $\boxed{\text{A}} < x < \boxed{\text{B}}$.

(ii) The range of values of a such that $|x - 2| < a$ is a necessary condition for $x^2 - 3x - 10 < 0$ is $\boxed{\text{C}}$.

(iii) The range of values of a such that $|x - 2| < a$ is a sufficient condition for $x^2 - 3x - 10 < 0$ is $\boxed{\text{D}}$.

$\textcircled{0}$ 2	$\textcircled{1}$ 5	$\textcircled{2}$ -2	$\textcircled{3}$ -5
$\textcircled{4}$ $a \geqq 2$	$\textcircled{5}$ $a \geqq 3$	$\textcircled{6}$ $a \geqq 4$	
$\textcircled{7}$ $0 < a \leqq 2$	$\textcircled{8}$ $0 < a \leqq 3$	$\textcircled{9}$ $0 < a \leqq 5$	

Q 4 Let d be the common difference of an arithmetic progression $\{a_n\}\,(n = 1,2,3, \cdots)$ which satisfies the two conditions

$$a_5 a_7 - a_4 a_9 = 60, \qquad a_{11} = 25.$$

Then

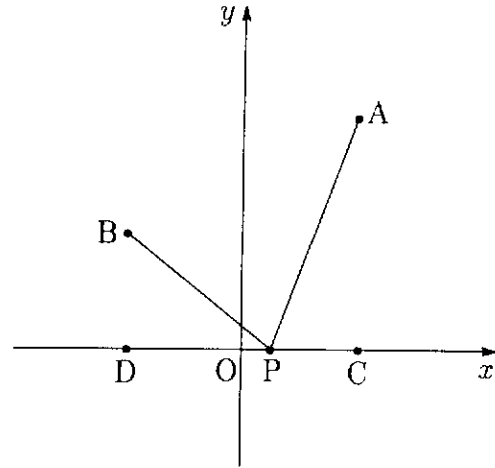(1) Either $d = \boxed{\text{A}}$ or $d = \boxed{\text{B}}$, where $\boxed{\text{A}} > \boxed{\text{B}}$.

(2) If $d = \boxed{\text{A}}$, then $a_1 = \boxed{\text{C}}$, $a_n = \boxed{\text{D}}\,n - \boxed{\text{E}}$, and the sum of the first n terms is 195 when $n = \boxed{\text{F}}$.

Q 5 Consider the four points

$$A(1,2), \quad B(-1,1), \quad C(1,0), \quad D(-1,0)$$

on the xy-plane. Let P be a point on the segment CD, excluding the endpoints. We are to find the point P at which $\angle APB$ is maximized.

Set $\alpha = \angle APC$, $\beta = \angle BPD$ and $\theta = \angle APB$, and let t be the x-coordinate of point P. Then

$$\tan \alpha = \frac{\boxed{\text{A}}}{\boxed{\text{B}} - t}, \quad \tan \beta = \frac{\boxed{\text{C}}}{\boxed{\text{D}} + t}$$

and hence

$$\tan \theta = \frac{t + \boxed{\text{E}}}{t^2 + \boxed{\text{F}}}.$$

When we differentiate the right-hand side of this equation with respect to t, we have

$$\left(\frac{t + \boxed{\text{E}}}{t^2 + \boxed{\text{F}}} \right)' = - \frac{t^2 + \boxed{\text{G}}\, t - 1}{(t^2 + \boxed{\text{H}})^2}.$$

Therefore the coordinates of point P are

$$\left(\boxed{\text{I}}, \ 0 \right).$$

Q 6 Let α be a real number. Let us translate the graph of the cubic function

$$y = f(x) = x^3 + ax^2 + bx + c \qquad \cdots\cdots\cdots \;\text{①}$$

so that the point $(\alpha, f(\alpha))$ on the graph of ① is translated into the origin $(0,0)$, and express the function of the translated graph in terms of $f'(\alpha)$ and $f''(\alpha)$.

First, we have

$$f'(\alpha) = \boxed{\;\textbf{A}\;}\,\alpha^2 + \boxed{\;\textbf{B}\;}\,a\alpha + b \qquad \cdots\cdots\cdots \;\text{②}$$

$$f''(\alpha) = \boxed{\;\textbf{C}\;}\,\alpha + \boxed{\;\textbf{D}\;}\,a. \qquad \cdots\cdots\cdots \;\text{③}$$

Next, we consider the translation which translates the point $(\alpha, f(\alpha))$ on the graph of ① into the origin, that is, we replace x with $x + \alpha$ and y with $y + f(\alpha)$ in ①, and obtain the expression

$$y = x^3 + \frac{f''(\alpha)}{\boxed{\;\textbf{E}\;}}\, x^2 + f'(\alpha)x$$

by using ② and ③.

As an example, consider the function

$$f(x) = x^3 - 12x^2 + 48x - 68. \qquad \cdots\cdots\cdots \;\text{④}$$

As

$$f'(\boxed{\;\textbf{F}\;}) = 0 \quad \text{and} \quad f''(\boxed{\;\textbf{G}\;}) = 0,$$

we see that when we translate the graph of ④ so that the point $(\boxed{\;\textbf{H}\;}, \boxed{\;\textbf{I}\;})$ on the graph is moved to the origin, we get the graph of $y = x^3$.

Q 7   Consider the curve $y = 2\log x$, where log is the natural logarithm. Let ℓ be the tangent to that curve which passes through the origin, let P be the point of contact of ℓ and that curve, and let m be the straight line perpendicular to the tangent ℓ at P. We are to find the equations of the straight lines ℓ and m and the area S of the region bounded by the curve $y = 2\log x$, the straight line m, and the x-axis.

Let t be the x-coordinate of the point P. Then t satisfies $\log t =\boxed{\textbf{A}}$. Hence the equation of ℓ is

$$y = \frac{\boxed{\textbf{B}}}{e}\,x.$$

The equation of m is

$$y = -\frac{e}{\boxed{\textbf{C}}}\,x + \frac{e^2}{\boxed{\textbf{D}}} + \boxed{\textbf{E}}.$$

Thus the area S of the region is

$$S = \boxed{\textbf{F}} + \frac{\boxed{\textbf{G}}}{e}.$$